ESERCIZI SUI PUNTI ESTREMI

TEST MATRICE HESSIANA

$\bar{x}_0$ P.TO STAZIONARIO INTERNO AL DOMINIO

- SE MAT $(H)_{\bar{x}_0}$ DEF. POS. → P.TO DI MIN.
- SE " " NEG. → " " MAX $\quad \forall_{n \geq 02}$
- " " INDEFINITA → " " SELLA

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i) CI INTERESSA IL SEGNO DEL DET (H)
NON IL VALORE

ii) $a = 0 \vee c = 0 \Rightarrow |A| = -b^2$ → < 0 se $b \neq 0$; → $= 0$ → CASO BASTARDO

ESERCIZIO

$f(x,y) = x^4 - 2(x^2 - y^2) - 4y$

$f_x = -4x^3 + 8xy \qquad S_f = \{A, B, C\}$

$f_y = 4x^2 - 4y - 4 \qquad A = (0,-1) \quad B = (-\sqrt{2}, 1)$

$C = (\sqrt{2}, 1)$

$f_{xx} = -12x^2 + 8y$

$f_{yy} = -4$

$f_{xy} = 8x$

$$H = \begin{bmatrix} -12x^2 + 8y & 8x \\ 8x & -4 \end{bmatrix}$$

$H_A > 0 \wedge a < 0$

$$H_A = \begin{bmatrix} 0 - 8 & 0 \\ 0 & -4 \end{bmatrix} = -8 \cdot -4 = 32 > 0$$

MAX

$$H_B = \begin{bmatrix} -12(-\sqrt{2})^2 + 8 & 8(-\sqrt{2}) \\ 8(-\sqrt{2}) & -4 \end{bmatrix} = |H| < 0 \to \text{SELLA}$$

$|H|_C < 0 \to$ SELLA

$|H| = 48x^2 - 32y - 64x^2$

$= -16x^2 - 32y$

$= -16(x^2 + 2y)$

SE $y > 0 \to x^2 + 2y > 0 \Rightarrow$ B E C SONO SELLA

PER MOSTRARE CHE NON AMMETTE ESTREMO GLOBALE:

$f(x,y) > f(A) \longrightarrow 1°$ APPROCCIO

2° : CERCO UN SOTTODOMINIO SUL QUALE LA f TENDE A $+\infty$ (STRONG)

1° APPROCCIO : $f(0,-1) = 2$

2° APP. : ASSE x, y, $y = x$, $y = -x$, $y = x^2$, $y = -x^2$

$$f(x,y) = x^4 - 2(x^2 - y)^2 - 4y \qquad f(A) = 2$$

$$y = x^2 \qquad x^4 - 2(0) - 4x^2 \xrightarrow{x \to +\infty} +\infty \quad \text{OK!} > 2$$

ESERCIZIO

$$f(x,y) = (\overset{\text{POS}}{x^2} + xy)\, \underbrace{e^{y-x}}_{\text{SEMPRE POS}}$$

$$f_x = e^{y-x}(2x + y - x^2 - xy) \qquad S_f = \{ \bar{0}, A \}$$

$$f_y = e^{y-x}(x^2 + xy + x) \qquad A = (1/2, -3/2)$$

2° PASSO

$$f_{xx} = [e^{y-x} \cdot -1] \cdot [2x + y - x^2 - xy] + [e^{y-x}] \cdot [2 - 2x - y]$$

$$= e^{y-x}(x^2 + xy - 4x - 2y + 2)$$

$$f_{yy} = e^{y-x}(x^2 + xy + x + x) = e^{y-x}(x^2 + xy + 2x)$$

$$f_{xy} = e^{y-x}(2x + y - x^2 - xy + 1 - x) = e^{y-x}(x + y - x^2 - xy + 1)$$

$$\tilde{H} = \begin{bmatrix} x^2 + xy - 4x - 2y + 2 & x + y - x^2 - xy + 1 \\ x + y - x^2 - xy + 1 & x^2 + xy + 2x \end{bmatrix}$$

$$H_{(0,0)} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = 2 \cdot 0 - 1 \cdot 1 = -1 < 0$$

$$\left.\begin{array}{l} \det(H_{00}) < 0 \\ a > 0 \end{array}\right\} \text{INDEFINITO : P.TO SELLA}$$

$$H_{(1/2,\,-3/2)} = \begin{bmatrix} 2-2x-y & 1-x \\ 1-x & x \end{bmatrix}$$

$$= \begin{bmatrix} 2-2\left(\frac{1}{2}\right)+\frac{3}{2} & 1-1/2 \\ 1-1/2 & 1/2 \end{bmatrix} \to \begin{bmatrix} 5/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \to |\tilde{H}(A)| = 1 > 0$$

$5/2 > 0 \to A$ P.TO DI MINIMO

ESERCIZIO

$$f(x,y) = x^4 - 6x^2y^2 + y^4$$

$$f_x = 4x^3 - 12xy^2$$

$$f_y = -12x^2y + 4y^3$$

$$\begin{cases} 4x^3 - 12xy^2 = 0 \\ -12x^2y + 4y^3 = 0 \end{cases} \quad \begin{cases} x^3 - 3xy^2 = 0 \\ -3x^2y + y^3 = 0 \end{cases} \quad \begin{cases} x(x^2-3y^2) = 0 \\ \end{cases}$$

$x(x^2-3y^2)=0$: $x = 0$ ①; $x^2 - 3y^2 = 0$ ② ✓

① $y^3 = 0 \to y = 0 \qquad A = (0,0)$

② $x^2 - 3y^2 = 0 \to x^2 = 3y^2 \to x = \pm\sqrt{3}\,y$

$-3 \cdot 3y^2 \cdot y + y^3 = 0 \to -9y^3 + y^3 = -8y^2 = 0 \to y = 0 \to B = (0,0)$

$S_f = \{\bar{0}\}$

2° PASSO

$f_{xx} = 12x^2 - 12y^2 \qquad f_{yy} = -12x^2 + 12y^2$

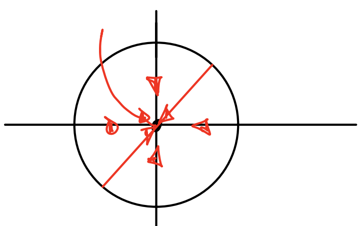
$f_{xy} = -24xy$

$$H = \begin{bmatrix} 12x^2 - 12y^2 & -24xy \\ -24xy & -12x^2 + 12y^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ MATRICE NULLA

SPERO SIA SELLA!

$$\Delta f_{\bar{0}}(h,k) = f(x_0+h, y_0+k) - f(x_0,y_0) = f(h,k) - f(0,0)$$



SE CAMBIA SEGNO IN BASE A COME MI AVVICINO $\Rightarrow$ SELLA

$$f(x,y) = x^4 - 6x^2y^2 + y^4$$

$$\Delta f_0 = f(h,k) = h^4 - 6h^2k^2 + k^4 =$$

$h=0$ asse y $\rightarrow f(0,k) = k^4 > 0$

$k=0$ asse x $\rightarrow f(h,0) = h^4 > 0$

$h=k$ $\qquad f(h,h) = h^4 - 6h^4 + h^4$

$= -4h^4 < 0$ sempre $\checkmark$ allora $(0,0)$ punto di sella

ESERCIZIO

$$f(x,y) = (y-1)^2 (x^2+y-1)$$

1) f ammette infinite p.ti stazionari

2) $A=(0,1)$ è di sella

3) $B=(1,1)$ è di min.

1° passo

$$f_x = (y-1)^2 \cdot 2x$$

$$f_y = 2(y-1)(x^2+y-1) + (y-1)^2 \cdot 1$$

$$= (y-1)(2x^2+3y-3)$$

$$\begin{cases} 2x(y-1)^2 = 0 \quad \begin{matrix} x=0 \ ① \\ y=1 \ ② \end{matrix} \\ (y-1)(2x^2+3y-3) = 0 \end{cases}$$

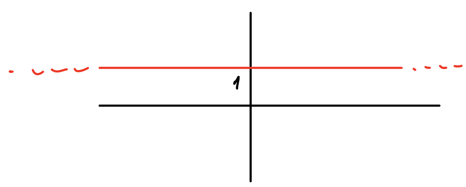
$x+y-x^2-xy+1$

① $(y-1)(3y-3) = 0 \rightarrow 3(y-1)^2 = 0 \rightarrow y=1 \qquad A = (0,1)$

② $y=1 \rightarrow 0=0 \checkmark$ con $y=1$, ho sol. infinite.

È sufficiente che $y=1$ indipendentemente dalla x

$$S_p = \{(x,y) \in \mathbb{R}^2 : y = 1\}$$



2° PASSO

$f_{xx} = 2(y-1)^2 \rightarrow$ SEMPRE ZERO

$f_{xy} = 2(y-1)2x$

$\rightarrow$ SEMPRE 0 CON $y=1$

SEMIDEFINITA

$$\Delta f_{(0,1)}(h,k) = f(x_0+h, y_0+k) - f(x_0, y_0)$$

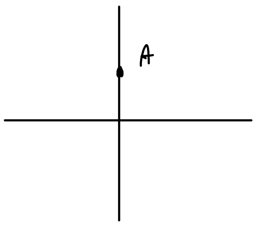
$$= f(h, 1+k) - f(0,1)$$

$$= f(h, 1+k) - 0$$

$$f(h, 1+k) = k^2(h^2+k) \rightsquigarrow h^2+k$$

$$\downarrow A$$

$$1 > 0$$



PER TH. PERMANENZA DEL SEGNO $h^2+k \rightarrow$ pos. IN TUTTO UN INTORNO DI A